COMPARISON OF DIFFERENT FRACTAL DIMENSION MEASURING ALGORITHMS FOR RE-TM M-O FILMS

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I. INTRODUCTION

As outlined in an earlier paper[1], noise in magneto-optical (M-O) recording devices is classed as that which is system-related (laser, electronic, and shot noise), and that which is media-related. Media noise is rooted in the magnetic and M-O properties of the recording media. The measure of the fractal dimension, D, for domain boundaries was proposed to investigate media noise and its relation to the microstructure and micromagnetics of thin films. Some of the possible sources of domain boundary jaggedness, and hence, different measures of D, might include structural/magnetic inhomogenieties as well as competition between domain wall energy and demagnetization.

Earler work concentrated on the conceptually simple divider method for measuring the fractal dimension, D, of binary images of M-O domains, both circular and line. To review this method, recall Richardson's[2] comparisons of the measurements of the coastlines of Europe. He discovered that when the boundary was measured with rulers of different lengths, substantially different values for the coastline length resulted. If the logarithm of the boundary as a function of ruler length was plotted versus the logarithm of ruler length, the points lay on a line of constant slope, m, that was related to D, the fractal dimension in the following way:

$$D = 1 - m.$$

However, certain films, notably, those which are nucleation-dominated, do not produce images that are particularly well conditioned for application of the divider method. Also, domain structure can occur within the interior of the major domain wall, and this is not measured by the divider method. A two-dimensional technique is clearly required. Two such 2-D techniques that can be employed are the amplitude spectrum method, and the box counting method.

The amplitude method (or power spectrum method) is based on Fourier methods. Using the same binary (or even grey-level) image, the 2-D FFT is computed, with only the amplitude of each frequency component of interest. Then, these frequency amplitudes are averaged for each integer frequency component to form an array of radial wave vectors. The log of amplitude components is then plotted versus the log of the radial wave vectors. A least squares line fit results provides the slope of the line, and D is calculated using the following relationship:

$$D = E + 3/2 + m$$

where E is the Euclidean dimension. The value of one (1) is used for the value of E so that this techniqe can be compared with the divider method.

The box counting method is probably the simplest technique of all. After the binary image is gotten, one slides squares with edge size ϵ through the image. If the box contains any of the domain pixels, the count for the number of boxes is incremented. A plot of the logarithm of image area as a function of box area versus the logarithm of the edge dimension is plotted, and a line fitted to determine its slope. The fractal dimension, D, is then calculated using the equation

$$D = 1 + m.$$

The box counting method was not used in this study.[3]

II. EXPERIMENT

For a group of seven (7) different amorphous, RE-TM magneto-optical thin films, fives images were recorded with different domain radii for each sample, ranging from approximately 40 μm to 120 μm. Then, the fractal dimension, D, for each expansion was measured and averaged using two measurement techniuqes: the divider method, and amplitude spectrum method. An additional metric was utilized, which is termed the local fractal dimension, or local D. In this measurement, a window is moved through the data, and then D is calculated for each window. That is, for a 10 component window, one calculates D for data points 1 through 10, then 2 through 11, 3 through 12, and so on. An average and standard deviation are then found for the data. Typically, the local D is used as a figure of merit to rank the goodness of one's fractal measuring algorithm, with a perfect result being a horizontal line. Here, it is used to compare the average D against that calculated for a specific range of data that was selected after examination of the log-log plot for both techniques. In this way, a less skewed measure of D might be possible, since the influence of the experimenter is greatly reduced. The data for these seven samples is summarized in Table 1. As can be seen in the table, there is good agreement between the local D, and that calculated for a narrow band of data. Figure 1 graphically compares the two approaches of narrowband and average of local D for the two fractal algorithms. In Fig. 1, the sample number corresponds to the numbering in

Table 1. Figure 2 shows representative images of each sample. Figures 3 and 4 are examples of the log-log plots for the divider method and amplitude spectrum method, respectively.

III. CONCLUSION

In general, it appears that either the divider technique or amplitude spectrum technique may be used interchangeably to measure the D inherent in domain wall structure of ideal images. However, some caveats must be observed for best results.

The divider technique is attractive for its simplicity and relatively modest computation requirements. But, it is sensitive to noise, in that noise pixels that touch the domain boundary are interpreted as being part of the boundary, skewing the measurment. Also, it is not useful in measuring nucleation-dominated films or domains that have significant amounts of structure within the interior of the domain wall.

The amplitude spectrum method is more complex, and less intuitive than the divider method, and somewhat more expensive to implement computationally. However, since the camera noise tends to be white, the noise can be avoided in the measurement of D by avoiding that portions of the curve that is flat (due to the white noise) when the least squares line is fit to the plot. Also, many image processing software packages include an FFT facility, while the user will most likely have to write his own edge extraction routine for the divider method. The amplitude spectrum method is a true two-dimensional technique that probes the interior of the domain wall, and in fact, can measure arbitrary clusters of domains. It can also be used to measure grey-level images, further reducing processing steps needed to threshhold the image.

REFERENCES

1. B.E. Bernacki and M. Mansuripur, *J. Apl. Phys.*, **69** (1), 1991.

2. B.B. Mandelbrot, *The Fractal Geometry of Nature* (W.H. Freeman, New York, 1983).

3. The box counting method is not sensitve to the structure in domain walls for binary images of domains that are predominately black. This is due to the small precentage of the total image that the domain wall represents, and thus is dwarfed by the dominant morphology, which is that of a round black domain.

Table 1

SAMPLE No.	Ruler (D) Local (D)	Spectral (D) Local (D)
1 12990G $Tb_{18.3}Fe_{74.5}Ar_{7.2}$	1.293 ± .027 1.279 ± .011	1.288 ± .022 1.309 ± .024
2 12490B $Tb_{28.1}Fe_{71.9}$	1.281 ± .040 1.284 ± .050	1.272 ± .047 1.318 ± .031
3 12990B $Tb_{24.1}Fe_{76.0}$	1.095 ± .005 1.086 ± .007	1.129 ± .011 1.113 ± .019
4 12990A $Tb_{22.5}Fe_{77.6}$	1.015 ± .006 1.016 ± .006	1.037 ± .003 1.045 ± .012
5 830 $Tb_{17.2}Fe_{60.3}Co_{7.4}Ar_{15.2}$	1.051 ± .006 1.052 ± .009	1.054 ± .009 1.081 ± .009
6 821 $Tb_{23.4}Fe_{57.6}Co_{8.5}Ar_{10.5}$	1.149 ± .014 1.151 ± .018	1.099 ± .011 1.134 ± .003
7 819 $Tb_{22.9}Fe_{58.4}Co_{9.4}Ar_{9.3}$	1.163 ± .024 1.187 ± .041	1.118 ± .030 1.188 ± .081

TABLE 1.

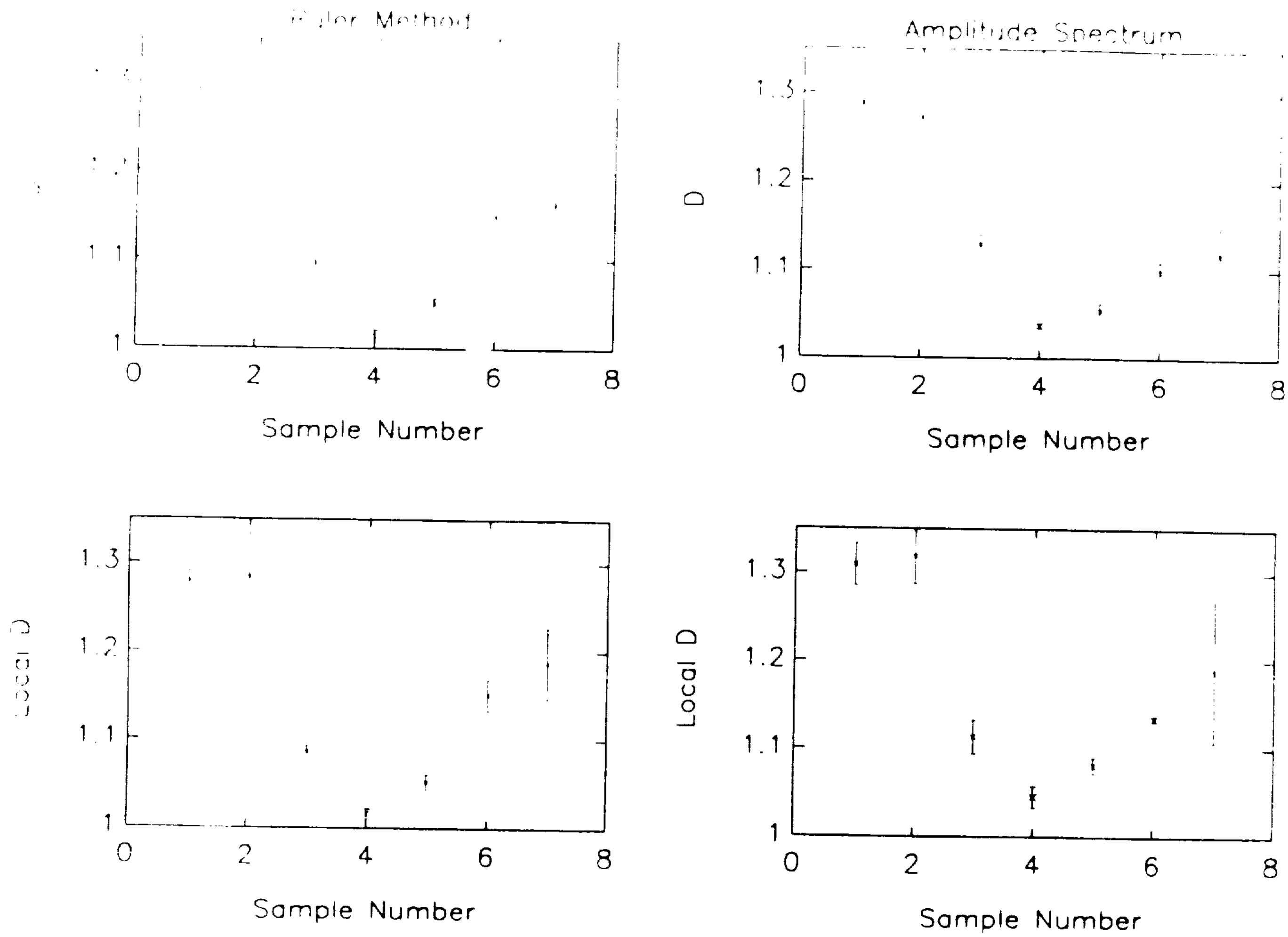

Fig. 1

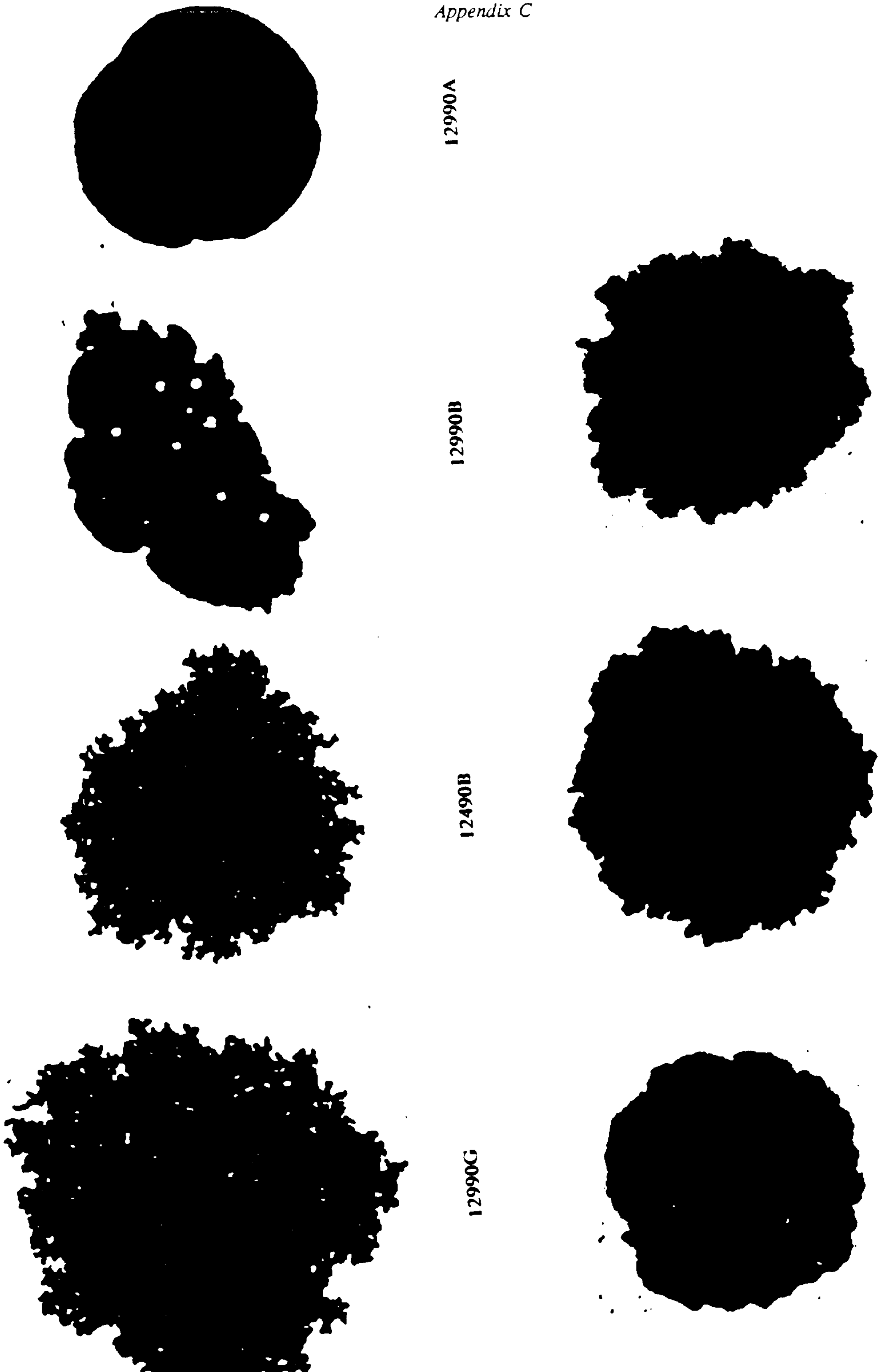

Figure 2. Example images of samples used in the comparison study.

C-2

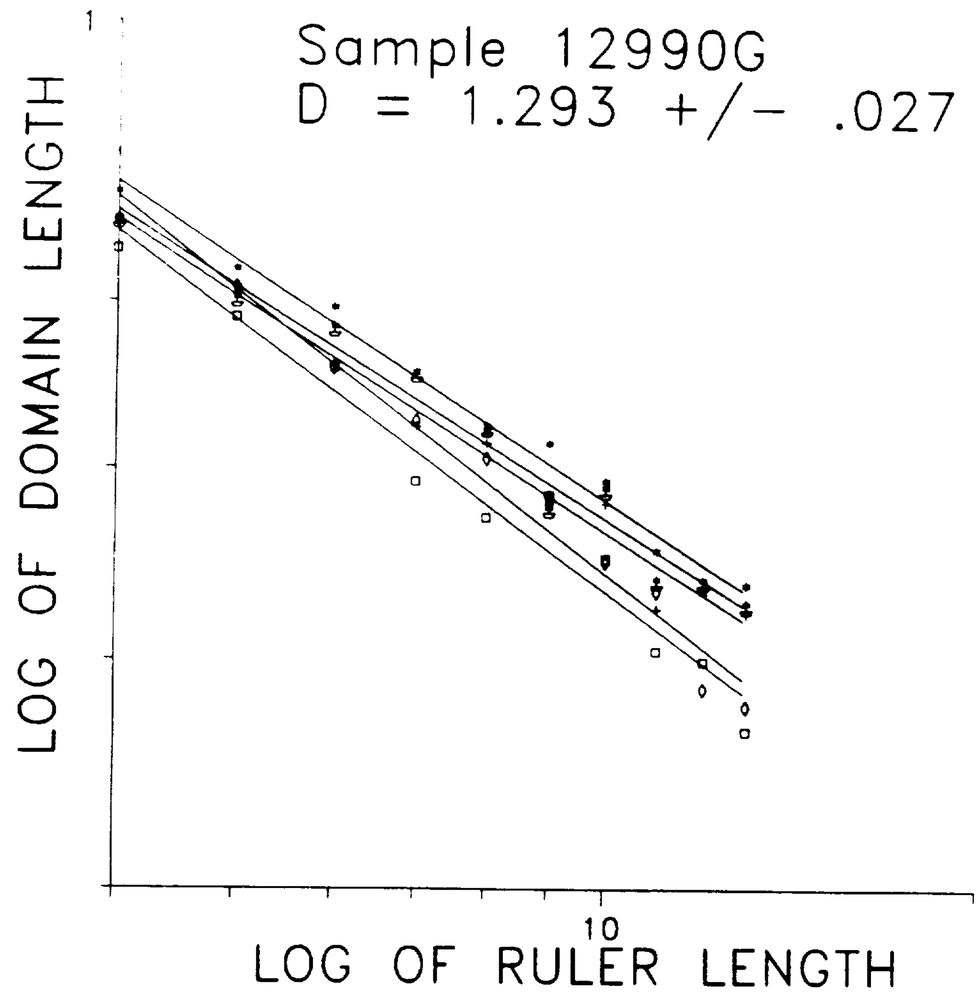
Sample 12990G
D = 1.293 +/- .027
1
LOG OF DOMAIN LENGTH
10
LOG OF RULER LENGTH

Fig. 3

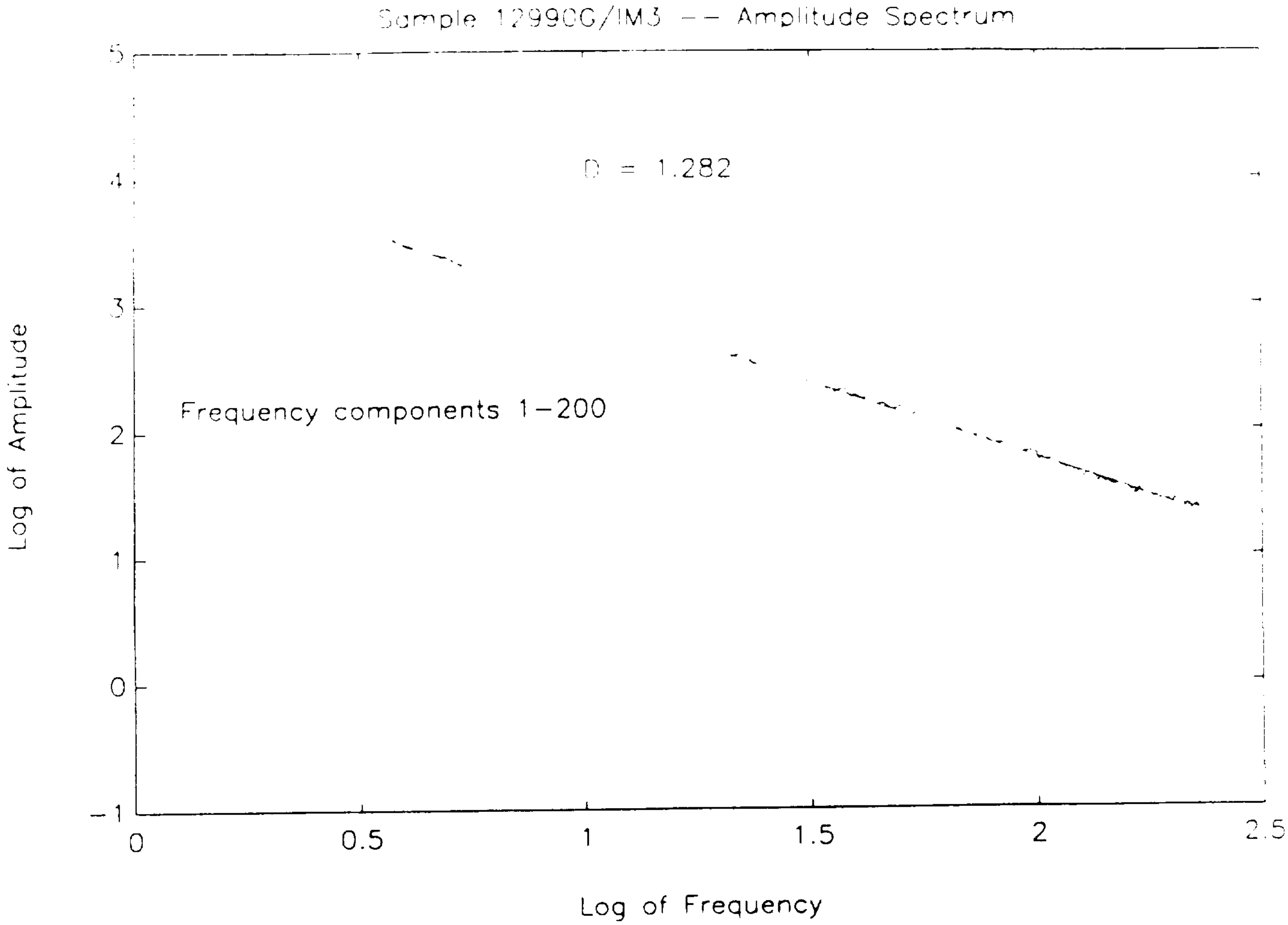

Fig. 4

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